Loss control regions in optimal control problems

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Abstract

In this presentation, we address optimal control problems involving *loss control regions*. In this context, the state space is partitioned into disjoint sets referred to as regions:

$$\mathbb{R}^n = \bigcup_{k=1}^{N} \overline{X_k},$$

where X_k are non-empty, open, disjoint sets. These regions are classified into two types: *control regions* and *loss control regions*. We introduce an index $q_k \in \{0, 1\}$ to distinguish between them as follows:

$$q_k = \begin{cases} 1 & \text{if } X_k \text{ is a control region,} \\ 0 & \text{if } X_k \text{ is a loss control region,} \end{cases} \qquad \text{for all } k = 1, \dots, N.$$

We consider a control system governed by a smooth dynamics $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. When the state belongs to a control region, the control is permanent (i.e., the control value can be modified at any time t). In this case, the control system is given by

$$\dot{x}(t) = f(x(t), u(t)), \qquad \text{if } q_k = 1.$$

On the other hand, when the state belongs to a loss control region, the control must remain constant. More precisely, it is equal to the last assigned value u_k before the state enters the loss control region, which is held until the state exits this region. In this case, the control system is

$$\dot{x}(t) = f(x(t), u_k), \qquad \text{if } q_k = 0.$$

The goal of this presentation is twofold. First, we derive a corresponding *Pontryagin maximum principle* (PMP, for short) using an *augmentation technique*, which allows us to reduce the original problem to a classical optimal control problem. Second, we propose a two-step numerical scheme to solve such optimal control problems. The approach uses a *direct method* that solves a regularized problem, followed by an *indirect method* (based on the corresponding PMP) that solves the original problem. Lastly, we apply this approach to several illustrative examples.

References

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3. T. Bayen, **A. Bouali**, L. Bourdin, and O. Cots, *On the reduction of a spatially hybrid optimal control problem into a temporally hybrid optimal control problem*, *Ivan Kupka Legacy: A Tour Through Controlled Dynamics*, 2024, vol. 12.